

Total No. of Questions : 8]  
(1126)

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**B.A./B.Sc. (General) IIIrd Semester (0003)  
Examination**

**0241**

**MATHEMATICS**

**Paper : I**

(Advanced Calculus-I)

**Time : 3 Hours]**

**[Maximum Marks : 30**

*Note :- Attempt five questions in all, selecting at least two questions from each Section.*

**Section-A**

1. (a) Show that :

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^4 y^4}{(x^2 + y^4)^3}$$

does not exist.

(b) Discuss the continuity of the function at  $(0, 0)$

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases} \quad 3,3$$

**A-66**

( 1 )

Turn Over

2. (a) If  $z = xy \tan \frac{y}{x}$ , prove that :

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 2z$$

(b) If  $z$  is a function of  $x$  and  $y$ , prove that if

$$x = e^u + e^{-v}, y = e^{-u} - e^v, \text{ then}$$

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$$

3. (a) Show that  $f(x, y) = \sin x + \sin y$  is differentiable at every point of  $\mathbb{R}^2$ .

(b) For  $\phi = x^2 y^3 z^4$ , find the directional derivative of  $\phi$  at  $(2, 3, 1)$  in the direction making equal angles with  $x$ ,  $y$  and  $z$ -axis.

4. (a) Find constants  $a$ ,  $b$  and  $c$  so that  $\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$  is irrotational.

(b) If  $\vec{A} = xyz \hat{i} + xz^2 \hat{j} - y^3 \hat{k}$  and

$\vec{B} = x^3 \hat{i} - xyz \hat{j} + x^2 z \hat{k}$ , then find :

$$\frac{\partial^2 \vec{A}}{\partial y^2} \times \frac{\partial^2 \vec{B}}{\partial x^2}$$

at (1, 1, 0).

3,3

### Section-B

5. (a) If  $H = f(x, y, z)$  is a homogeneous functions of  $x, y$  and  $z$  of degree  $n$  then :

$$x \frac{\partial H}{\partial x} + y \frac{\partial H}{\partial y} + z \frac{\partial H}{\partial z} = nH$$

(b) Use Taylor's theorem to expand  $xy^2 + 3x - 2$  in powers of  $(x + 2)$  and  $(y - 1)$ . 3,3

6. (a) If  $x = r \cos \theta \cos \varphi$ ,  $y = r \sin \theta \sqrt{1 - m^2 \sin^2 \varphi}$  and  $z = r \sin \varphi \sqrt{1 - n^2 \sin^2 \theta}$ , where  $m^2 + n^2 = 1$ , then show that :

$$\frac{\partial(x, y, z)}{\partial(r, \theta, \varphi)} = \frac{r^2 (m^2 \cos^2 \varphi + n^2 \cos^2 \theta)}{\sqrt{1 - m^2 \sin^2 \varphi} (1 - n^2 \sin^2 \theta)}$$

A-66

( 3 )

Turn Over

(b) Show that the functions  $u = x^3 + x^2y + x^2z - z^2(x + y + z)$ ,  $v = z + x$ ,  $w = x^2 - z^2 + xy - zy$  are not independent of one another. Also find the relation between them.

7. (a) Find the envelope of the family of lines  $x \cos^3 \theta + y \sin^3 \theta = a$ ,  $\theta$  being the parameter.

(b) Find the evolute of the parabola  $y^2 = 4ax$  regarding it as an envelope of its normal.

8. (a) Find the maximum and minimum values of the function :

$$f(x, y) = \sin x + \sin y + \cos(x + y)$$

(b) Let  $f : \mathbb{R}^3 \rightarrow \mathbb{R}$  be defined by  $f(x, y, z) = xyz$ .

Determine  $x, y, z$  for maximum of  $f$  subject to condition  $xy + 2yz + 2zx = 108$ .

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